

## Test in Mathematics THE PROGRAM

### I. Basic concepts.

Natural numbers. Divisibility. Prime and composite numbers. Greatest common divisor and Least common multiple. Integer, rational and real numbers. Percent. Absolute value (modulus) of a real number, degree, roots, square root, logarithm. Sine, cosine, tangent, and cotangent of an angle (of a number). Arcsine, arccosine, arctangent, arccotangent of a number. Numerical and symbolic expressions. Equalities and identities. Function, its domain and range. Function increase, decrease, periodicity. Odd and even functions, derivative. maxima and minima, graph of a function. Linear, quadratic, powers, exponential, logarithmic, trigonometric functions. Equations, inequalities, systems of the equations. Solution of equation, inequality, system of equations. Equivalence. Arithmetic and geometric progressions. Straight line on a plane. Beam, segment, polygonal chain, angle. Triangle. Median, bisector, height. A convex polygon. Square, rectangle, parallelogram, rhomb, trapeze. equilateral polygon. Diagonal. Circle. Radius, chord, diameter, tangent, secant. Arc, sector. Central and inscribed angle. Straight line and plane in space. Dihedral angle. Polyhedra. Cube, parallelepiped, prism, pyramid. Cylinder, cone, sphere. Equality and similarity of figures. Symmetry. Parallelism and perpendicularity of straight lines, planes. Skew straight lines. Angle between straight lines, planes, a straight line and a plane. Tangency. The inscribed and circumscribed figures on a plane and in space. Cross section of a body with a plane. Measure of angle. Length of a segment, a circle and an arc. The area of a polygon, circle and circular sector. The area of a surface and volume of a polyhedron, a cylinder, a cone, a sphere. Coordinate straight line. Numerical intervals. The Cartesian coordinates on a plane and in space. Vectors.

### II. The theoretical part

#### Algebra and the analysis.

Divisibility rules for 2, 3, 5, 9, 10. Properties of the numerical inequalities. Mathematical identities. Properties of linear function and its graph. Quadratic formula. Quadratic factorization. Viète's formulas. Properties of quadratic function and its graph. The inequality between arithmetic and geometric means of two numbers.

The inequality for the sum of two mutually inverse numbers. Formulas of the general term and of the sum of first  $n$  members of an arithmetic progression. Formulas of the general term and of the sum of first  $n$  members of a geometrical progression. Properties of natural and integer powers. Properties of the principal  $n$ -th root. Properties of rational powers. Properties of the power function with integer exponent and its graph. Properties of exponential function and its graph. Definition of a logarithm. Logarithm of product, powers, fractions. Change of base. Properties of a logarithmic function and its graph. Concept of function. Derivative. Geometrical sense of a derivative. A finding of a maxima and a minima of a function. Pythagorean trigonometric identity. Expression of trigonometric function in terms of other. Shifts and periodicity formulas. Angle sum and difference identities. Double- and half-angle formulas. Tangent half-angle formulas. Product-to-sum and sum-to-product identities. Phase-shift transform for linear combinations  $a \cdot \sin(x) + b \cdot \cos(x)$ . Solving of the basic trigonometrical equations. Properties of trigonometrical functions and their graphs.

#### Geometry.

Theorems of parallel straight lines on a plane. Properties of vertical and supplementary angles. Properties of an isosceles triangle. Sufficient conditions for a pair of triangles to be congruent. The sum of internal angles of a triangle. Exterior angle theorem. Properties of an midsegment of a triangle. Thales theorem. Sufficient conditions for a pair of triangles to be similar. Right

triangles. Pythagorean theorem. Property of a perpendicular bisector. Property of an angle bisector. Theorems about intersection of the altitudes, angle bisectors and medians. Property of the segments the angle bisector divides the opposite side into. Tangent lines to a circle. Equality of tangents to a circle, drawn from one point. Inscribed angle theorem. The theorem of an angle between a tangent and a chord. Chord-chord and secant-secant power theorems. Secant-tangent theorem. Property of the inscribed and circumscribed triangles. Property of the inscribed and circumscribed quadrilaterals. The sine and cosine rules. The sum of interior angles of a convex polygon. Sufficient conditions for a quadrilateral to be a parallelogram. Properties of a parallelogram. Properties of the midsegment of a trapezoid. The Euclidean distance between two points of the plane with Cartesian coordinates. The equation of the circle. Theorems of parallel straight lines in space. Conditions for a parallelism of a straight line and plane and for a parallelism of planes. A perpendicularity of a straight line and plane. The theorem of the common perpendicular to skew straight lines. Perpendicularity of planes. The theorem of the three perpendiculars.

### III. Requirements

In additional examination in mathematics the knowledge of the following can be checked up.

- Calculations with numbers, transformations of polynomials, fractions, expressions, containing power, exponential, logarithmic, and trigonometric functions.
- Graphs of linear, quadratic, power, exponential, logarithmic, and trigonometric functions.
- solving of equations and inequalities of degree 1 and 2, containing power, exponential, logarithmic, and trigonometric functions.
- solving of word problems
- geometrical constructions;
- use of an algebraic methods to problem solving in geometry and vice versa;
- use derivative for exploring and graphing functions